

# ON THE POWER SERIES OF DENEFF AND LOESER'S MOTIVIC VANISHING CYCLES OF JET POLYNOMIALS

(JOINT WORK WITH ARTHUR FOREY)

KIEN HUU NGUYEN

ABSTRACT. Let  $f$  be a non-constant polynomial in  $n$  variables over a field  $k$  of characteristic 0. Denef and Loeser introduced the notion of motivic vanishing cycles of  $f$  as an element in the localization  $\mathcal{M}_k^{\hat{\mu}}$  of the Grothendieck ring  $K_0(\text{Var}_k)$  of  $k$ -varieties with a good action of  $\hat{\mu} := \varprojlim_m \mu_m$  by inverting the affine line equipped with the trivial action of  $\hat{\mu}$ , where  $\mu_m$  is the group scheme over  $k$  of  $m^{\text{th}}$  roots of unity. In particular, if  $k$  is the field of complex numbers then Denef and Loeser showed that their motivic vanishing cycles and the complex  $\phi_f[n-1]$  has the same Hodge characteristic, where  $\phi_f$  is the complex of vanishing cycles in the usual sense. Motivated by the Igusa conjecture for exponential sums and the strong monodromy conjecture, we introduce the notion of Poincaré series of Denef-Loeser's vanishing cycles of jet polynomials of  $f$ , where jet polynomials of  $f$  are polynomials appearing naturally when we compute the jet schemes of  $f$ . By using Davison-Meinhardt's conjecture which was proved by Nicaise and Payne in 2019, we can show that our Poincaré series is a rational function over a quotient ring of  $\mathcal{M}_k^{\hat{\mu}}$  by very natural relations. In particular, we can recover Denef and Loeser's motivic vanishing cycles from our Poincaré series. Moreover, we can show that our Poincaré series owns a universal property in the sense that if  $k$  is a number field then the Igusa local zeta functions, the motivic Igusa zeta functions, the Poincaré series of exponential sums modulo  $p^m$  of  $f$  can be obtained from our Poincaré series by suitable specialization maps preserving the rationality. If time permits, I will present some initial consequences that have arisen during the study of our Poincaré series.

NORMANDIE UNIVERSITÉ, UNIVERSITÉ DE CAEN NORMANDIE - CNRS, LABORATOIRE DE MATHÉMATIQUES NICOLAS ORESME (LMNO), UMR 6139, 14000 CAEN, FRANCE

*Email address:* `huu-kien.nguyen@unicaen.fr`